4.1.5 Show that the family of regular languages is closed under finite union and intersection, that is, if $L_1$, $L_2$, ..., $L_n$ are regular, then $L_U = L_1 \cup L_2 \cup \cdots \cup L_n$ and $L_I = L_1 \cap L_2 \cap \cdots \cap L_n$ are also regular.
4.1.6 The **symmetric difference** of two sets $S_1$ and $S_2$ is defined as $S_1 \ominus S_2 = \{x : x \in S_1 \text{ or } x \in S_2, \text{ but } x \text{ is not in both } S_1 \text{ and } S_2\}$.

Show that the family of regular languages is closed under symmetric difference.
4.1.8 The complementary or (cor) of two languages by
\[ \text{cor}(L_1, L_2) = \{ w : w \in \overline{L_1} \text{ or } w \in \overline{L_2} \}. \]
Show that the family of regular languages is closed under the cor operation.
4.1.9 Which of the following are true for all regular languages and all homomorphisms? (a) $h(L_1 \cup L_2) = h(L_1) \cup h(L_2)$ (b) $h(L_1 \cap L_2) = h(L_1) \cap h(L_2)$ (c) $h(L_1 L_2) = h(L_1) h(L_2)$
4.1.13 If L is a regular language, prove that \( L_1 = \{uv : u \in L, |v| = 2\} \) is also regular, \( L, L_1 \subseteq \{a, b\}^* \).
4.1.17 The tail of a language is defined as the set of all suffixes of its strings, that is tail(L) = \{y : xy \in L \text{ for some } x \in \Sigma^* \}. Show that if L is regular, so it tail(L).
4.1.25 The min of a language $L$ is defined as $\min(L) = \{w \in L : \text{there is no } u \in L, v \in \Sigma^+, \text{ such that } w = uv\}$. Show that the family of regular languages is closed under the min operation.
4.2.3 Show that there exists an algorithm for determining if $\lambda \in L$, for any regular language $L$. 
4.2.6 Exhibit an algorithm for determining whether or not a regular language $L$ contains any string $w$ such that $w^R \in L$. 
4.2.8 Exhibit an algorithm that, given any regular language $L$, determines whether or not $L = L^*$. 
4.2.13 Show that there exists an algorithm that can determine for every regular language L, whether or not $|L| \geq 5$. 
4.2.14 Find an algorithm for determining whether a regular language $L$ contains an infinite number of even-length strings.
4.3.3 Show that the language $L = \{w : n_a(w) = n_b(w)\}$ is not regular. Is $L^*$ regular?
4.3.4 b) Prove that $L = \{a^n b^l a^k : k \neq n + l \}$ is not regular.
4.3.4 c) Prove that \( L = \{a^n b^l a^k : n = l \text{ or } l \neq k \} \) is not regular.
4.3.4 d) Prove that \( L = \{a^n b^l : n \leq l\} \) is not regular.
4.3.5 d) Determine whether or not \( L = \{a^n : n = 2^k \text{ for some } k \geq 0\} \) on \( \Sigma = \{a\} \) is regular.
4.3.6 b) Determine whether or not $L = \{a^n b^n : n \geq 1\} \cup \{a^n b^{n+2} : n \geq 1\}$ is regular.
4.3.8 Show that the language $L = \{a^n b^{n+k} : n \geq 0, k \geq 1\} \cup \{a^{n+k} b^n : n \geq 0, k \geq 3\}$ is not regular.
4.3.11 Show that the language $L = \{a^{n!} : n \geq 1\}$ is not regular.
4.3.13 Show that the following language is not regular $L = \{a^n b^k : n > k\} \cup \{a^n b^k : n \neq k - 1\}$. 
4.3.15 f) Make a conjecture whether or not \( L = \{a^n b^l : n \geq 100, l \leq 100\} \) is regular. Then prove your conjecture.
4.3.16 Is the following language regular? \( L = \{ w_1cw_2 : w_1, w_2 \in \{a, b\}^*, w_1 \neq w_2 \} \).
4.3.20 Is the following language regular? \( L = \{ww^Rv: v, w \in \{a, b\}^+\} \).
4.3.24 Suppose that we know that $L_1 \cup L_2$ and $L_1$ are regular. Can we conclude from this that $L_2$ is regular?
4.3.26 Let $L = \{a^n b^m : n \geq 100, m \leq 50\}$. (a) Can you use the pumping lemma to show that $L$ is regular? (b) Can you use the pumping lemma to show that $L$ is not regular? Explain your answers.
A4-1 a) State whether \( \{a^ib^j : i, j \geq 0 \text{ and } i + j = 5\} \) is regular or not and prove your answer.
A4-1 b) State whether \{a^i b^j : i, j \geq 0 \text{ and } i - j = 5\} is regular or not and prove your answer.
A4-1 c) State whether \( \{a^i b^j : i, j \geq 0 \text{ and } |i - j| = 0 \text{ (mod 5)} \} \) is regular or not and prove your answer.
A4-1 d) State whether \( \{w \in \{0,1,#\}^* : w = x#y, \text{ where } x, y \in \{0,1\}^* \text{ and } |x| \cdot |y| = 0(\text{mod } 5) \} \) is regular or not and prove your answer.
A4-1 e) State whether \( \{a^i b^j : 0 \leq i < j < 2000\} \) is regular or not and prove your answer.
A4-1 f) State whether \( \{w \in \{Y,N\}^* : w \text{ contains at least two Y's and at most two N's} \} \) is regular or not and prove your answer.
A4-1 g) State whether \( \{ w = xy : x, y \in \{a,b\}^* \text{ and } |x| = |y| \text{ and } n_a(x) \geq n_a(y) \} \) is regular or not and prove your answer.
A4-1 h) State whether \( \{ w = xyzy^R x : x, y, z \in \{a,b\}^* \} \) is regular or not and prove your answer.
A4-1 i) State whether \( \{ w = xyzy : x, y, z \in \{0,1\}^* \} \) is regular or not and prove your answer.
A4-1 j) State whether \( \{w \in \{0,1\}^* : n_0(w) \neq n_1(w)\} \) is regular or not and prove your answer.
A4-1 l) State whether \( \{ w \in \{a,b\}^* : \exists x \in \{a,b\}^+, w = xx^R x \} \) is regular or not and prove your answer.
A4-1 m) State whether \( \{ w \in \{a,b\}^* : \) the number of occurrences of the substring ab equals the number of occurrences of the substring ba} is regular or not and prove your answer.
A4-1 n) State whether \( \{ w \in \{a,b\}^* : w \text{ contains exactly two more } b \text{'s than } a \text{'s} \} \) is regular or not and prove your answer.
A4-1 o) State whether \( \{ w \in \{a,b\}^* : w = xyz, |x| = |y| = |z|, \text{ and } z = x \text{ with every } a \text{ replaced by } b \text{ and every } b \text{ replaced by } a \} \) is regular or not and prove your answer.

Example: \( \text{abbbabbaa} \in L \), with \( x = \text{abb}, y = \text{bab}, \text{ and } z = \text{baa} \).
A4-1 p) State whether \( \{w : w \in \{a-z\}^* \text{ and the letters of } w \text{ appear in reverse alphabetical order}\} \) is regular or not and prove your answer. For example, \( \text{spoonfeed} \in L \)
A4-1 q) State whether \( \{w : w \in \{a-z\}^* \text{ and every letter in } w \text{ appears at least twice} \} \) is regular or not and prove your answer.
For example, \( \text{unprosperousness} \in L \)